NOTE:
THE TEMPLATE IN THIS BOX IS ONLY USED TO INTERPRET FUNCTION STATEMENTS.
IT IS NOT USED TO TRANSLATE DERIVATIVE STATEMENTS.

A MECHANICAL WAY TO INTERPRET A FUNCTION STATEMENT

If $y = f(x)$, then you can start interpreting the statement $f(c) = d$ using the template:

> When **[*x*]** is **[*c*]** **[*units of x*]**,
> **[*y*]** is **[*d*]** **[*units of y*]**.

where all the parts in [ ] are filled in with their meaning or value from the actual situation.

EXAMPLE

If the time to come to a complete stop is a function of the force on the brakes, that is,
$t = f(b)$ where

> t is the stopping time, in seconds and
> b is the force applied to the brakes, in pounds,

then, to interpret $f(50) = 11$,

> **[*x*]** is "**force on the brakes**"
> **[*units of x*]** is "**pounds**"
> **[*y*]** is "**stopping time**"
> **[*units of y*]** is "**seconds**"
> **[*c*]** is "**50**"
> **[*d*]** is "**11**"

the template

When **[*x*]** is **[*c*]** **[*units of x*]**, **[*y*]** is **[*d*]** **[*units of y*]**.	**BECOMES**	When **force on the brakes** is **50** **pounds**, **stopping time** is **11** **seconds**.

This usually results in sentences that sound like a bad translation.

So, you should rewrite it into something more natural sounding.

> When a force of 50 pounds is applied to the brakes,
> it takes 11 seconds to come to a complete stop.

A MECHANICAL WAY TO INTERPRET A DERIVATIVE STATEMENT

The derivative of a function with a certain input and a certain output
tells you the rate at which the output of the function changes as its input changes.
If the output increases in value as the input increases in value, then the derivative is positive.
If the output decreases in value as the input increases in value, then the derivative is negative.

If $y = f(x)$, then you can start interpreting the statement $f'(c) = d$ using the template:

> When **[*x*]** is **[*c*]** **[*units of x*]**,
> **[*y*]** changes by **[*d*]** **[*units of y*]**
> per **[*units of x*]** increase in **[*x*]**.

where all the parts in [ ] are filled in with their meaning or value from the actual situation.

EXAMPLE

If the time to come to a complete stop is a function of the force on the brakes, that is,
$t = f(b)$ where

> t is the stopping time, in seconds and
> b is the force applied to the brakes, in pounds,

then, to interpret $f'(50) = -0.5$,

> **[*x*]** is "**force on the brakes**"
> **[*units of x*]** is "**pounds**"
> **[*y*]** is "**stopping time**"
> **[*units of y*]** is "**seconds**"
> **[*c*]** is "**50**"
> **[*d*]** is "**–0.5**"

the template

When **[*x*]** is **[*c*]** **[*units of x*]**, **[*y*]** changes by **[*d*]** **[*units of y*]** per **[*units of x*]** increase in **[*x*]**.	**BECOMES**	When **force on the brakes** is **50 pounds**, **stopping time** changes by **–0.5 seconds** per **pound** increase in **force on the brakes**.

This results in sentences that sound like a bad translation.

So, you should rewrite it into something more natural sounding.

> When a force of 50 pounds is applied to the brakes,
> the stopping time decreases by 0.5 seconds for each additional pound of force applied to the brakes.

NOTE

Because derivatives are instantaneous rates of change,
the statement above is only true at the input value specified.

So, if the force of 50 pounds is increased to 51 pounds,
it doesn't mean the stopping time decreases by exactly 0.5 seconds,
because the change in the force also probably results in a change in the derivative.

So, it might be more appropriate to say the stopping time decreases by approximately 0.5 seconds.
In fact, this is how derivatives are often used, to approximate a change in one quantity when another quantity changes.
For example, if the force on the brakes increases 3 pounds (to 53 pounds), the stopping time decreases by approximately 3*0.5 = 1.5 seconds.
(This would be the best approximation we could come up with based only on the derivative information.)

ADDITIONAL EXAMPLE 1

If the cost of repairing the damage caused by a baseball hitting a parked car depends on the velocity at which the baseball hits the car, that is, $C = f(v)$ where

C is the repair cost, in dollars ($) and
v is the velocity of the baseball, in feet per second (ft/s),

then, to interpret $f'(20) = 30$,

[*x*] is "**velocity of baseball**"
[*units of x*] is "**ft/s**"
[*y*] is "**cost of repair**"
[*units of y*] is "**$**"
[*c*] is "**20**"
[*d*] is "**30**"

the template

When [*x*] is [*c*] [*units of x*], [*y*] changes by [*d*] [*units of y*] per [*units of x*] increase in [*x*].	**BECOMES**	When **velocity of baseball** is **20 ft/s**, **cost of repair** changes by **30 $** per **ft/s** increase in **velocity of baseball**,

which we write more naturally as

When a baseball hits a parked car at 20 ft/s,
the cost of repair increases by $30 for each ft/s faster the ball hits the car.

ADDITIONAL EXAMPLE 2

If the number of iPhone applications sold per day by a certain vendor depends on the price of each application, that is, $A = f(p)$ where

A is the number of applications sold per day, and
p is the price per application, in dollars ($),

then, to interpret $f'(10) = -5$,

[*x*] is "**price per application**"
[*units of x*] is "**$**"
[*y*] is "**number of applications sold per day**"
[*units of y*] is ""
[*c*] is "**10**"
[*d*] is "**–5**"

the template

When [*x*] is [*c*] [*units of x*], [*y*] changes by [*d*] [*units of y*] per [*units of x*] increase in [*x*].	**BECOMES**	When **price per application** is **10 $**, **number of applications sold per day** changes by **–5** per **$** increase in **price per application**,

which we write more naturally as

When applications are sold at $10 each,
daily sales will drop by 5 applications for each dollar that the price is raised.

ADDITIONAL EXAMPLE 3

If your score on a test depends on how much time you spent studying for it the 24 hours just before the test., that is, $P = f(s)$ where

P is your score on the test, in points, and
s is the amount of time you studied the day before, in hours,

then, to interpret $f'(4) = 7$,

[*x*] is "**amount of time you studied the day before**"
[*units of x*] is "**hours**"
[*y*] is "**your score on the test**"
[*units of y*] is "**points**"
[*c*] is "**4**"
[*d*] is "**7**"

the template

When **[*x*]** is **[*c*]** **[*units of x*]**,
[*y*] changes by **[*d*]** **[*units of y*]**
per **[*units of x*]** increase in **[*x*]**.

BECOMES

When **amount of time you studied the day before** is **4** **hours**,
your score on the test changes by **7** **points**
per **hour** increase in **amount of time you studied the day before**,

which we write more naturally as

If you study for 4 hours the day before a test,
your score would go up by 7 points for each additional hour you studied that day.

ADDITIONAL EXAMPLE 4

If the amount you spend on light bulbs each year depends on how many hours a day you use them, that is, $A = f(u)$ where

A is the amount you spend on lightbulbs each year, in dollars, and
u is the daily usage of the lightbulb, in hours,

then, to interpret $f'(5) = 10$,

[*x*] is "**daily usage of lightbulbs**"
[*units of x*] is "**hours**"
[*y*] is "**amount you spend on lightbulbs each year**"
[*units of y*] is "**dollars**"
[*c*] is "**5**"
[*d*] is "**10**"

the template

When **[*x*]** is **[*c*]** **[*units of x*]**,
[*y*] changes by **[*d*]** **[*units of y*]**
per **[*units of x*]** increase in **[*x*]**.

BECOMES

When **daily usage of lightbulbs** is **5** **hours**,
amount you spend on lightbulbs each year changes by **10** **dollars**
per **hour** increase in **daily usage of lightbulbs**,

which we write more naturally as

If you use lightbulbs 5 hours a day,
you will spend $10 more each year on lightbulbs for every additional hour you use them each day.

ADDITIONAL EXAMPLE 5

If the lifespan of a light bulb depends on how many hours a day you use it, that is,
$L = f(u)$ where

L is the lifespan of a lightbulb, in months, and
u is the daily usage of the lightbulb, in hours,

then, to interpret $f'(5) = -1$,

[*x*] is "**daily usage of a lightbulb**"
[*units of x*] is "**hours**"
[*y*] is "**lifespan of the lightbulb**"
[*units of y*] is "**months**"
[*c*] is "**5**"
[*d*] is "**–1**"

the template

When **[*x*]** is **[*c*]** **[*units of x*]**, **[*y*]** changes by **[*d*]** **[*units of y*]** per **[*units of x*]** increase in **[*x*]**.	**BECOMES**	When **daily usage of a lightbulb** is **5** **hours**, **lifespan of the lightbulb** changes by **–1** **months** per **hour** increase in **daily usage of a lightbulb**,

which we write more naturally as

If you use a lightbulb 5 hours a day,
it will last one month less for every additional hour you use it each day.

ADDITIONAL EXAMPLE 6

If the time required for a disc of liquid metal to solidify depends on the thickness of the disc, that is,
$t = f(d)$ where

t is the time required for a disc to solidify, in minutes, and
d is the thickness of the disc, in millimeters,

then, to interpret $f'(2) = 6$,

[*x*] is "**thickness of a disc of liquid metal**"
[*units of x*] is "**mm**"
[*y*] is "**time required to solidify**"
[*units of y*] is "**minutes**"
[*c*] is "**2**"
[*d*] is "**6**"

the template

When **[*x*]** is **[*c*]** **[*units of x*]**, **[*y*]** changes by **[*d*]** **[*units of y*]** per **[*units of x*]** increase in **[*x*]**.	**BECOMES**	When **thickness of a disc of liquid metal** is **2** **mm**, **time required to solidify** changes by **6** **minutes** per **mm** increase in **thickness of a disc of liquid metal**,

which we write more naturally as

A 2mm thick disc of liquid metal will take 6 minutes longer to solidify for each extra millimeter in thickness.

ADDITIONAL EXAMPLE 7

If the value of a bottle of wine depends on its Wine Spectator rating, that is,
$v = f(r)$ where

v is the value of the wine, in dollars, and
r is the Wine Spectator rating of the wine, in points,

then, to interpret $f'(87) = 4$,

[*x*] is "**Wine Spectator rating of the wine**"
[*units of x*] is "**points**"
[*y*] is "**value of the wine**"
[*units of y*] is "**$**"
[*c*] is "**87**"
[*d*] is "**4**"

the template

When **[*x*]** is **[*c*]** **[*units of x*]**,
[*y*] changes by **[*d*]** **[*units of y*]**
per **[*units of x*]** increase in **[*x*]**.

BECOMES

When **Wine Spectator rating of the wine** is **87** **points**,
value of the wine changes by **4** **$**
per **point** increase in **Wine Spectator rating of the wine**,

which we write more naturally as

A wine which has a Wine Spectator rating of 87 points will go up in value by $4 for each additional point it receives from Wine Spectator.

CHECKING YOUR UNDERSTANDING

[a] If the output of a function decreases as the input of the function decreases, what can you say about the derivative ?
If the output of a function increases as the input of the function decreases, what can you say about the derivative ?

[b] $t = f(d)$ where d is the distance a car is driven each day, in miles, and
t is the time between oil changes, in months

What are the units of $f'(20)$? Is $f'(20)$ positive or negative ? Is $f'(70)$ positive or negative ?
What does $f'(30) = -0.1$ mean ?

[c] $m = f(r)$ where m is the mass of a 5 millimeter tall iron disk, in grams, and
r is the radius of the disk, in centimeters

What are the units of $f'(20)$? Is $f'(20)$ positive or negative ? Is $f'(70)$ positive or negative ?
Which has a larger size, $f'(20)$ or $f'(70)$?
What does $f'(10) = 250$ mean ?

[d] $e = f(s)$ where e is the fuel efficiency of a car, in miles per gallon, and
s is the speed at which the car is driven, in miles per hour

What are the units of $f'(20)$? Is $f'(20)$ positive or negative ? Is $f'(70)$ positive or negative ?
What does $f'(40) = 0.2$ mean ?